3.2 Vectors and Straight Lines

Problems Worksheet

Step Up In
Mathematics Specialist
Unit 3 and 4

1. Write the vector equation of the following lines in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$.

   a. Through points located at position vectors $A\ (\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})$ and $B\ (5\boldsymbol{i} - \boldsymbol{k})$.

   b. Through $A\ (-2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k})$ and perpendicular to the vector $\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$.

   c. Perpendicular to the line $\boldsymbol{r}.\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = 8$ and through $A\ (\boldsymbol{j} - 2\boldsymbol{k})$.

2. Determine the vector equation of the following straight-line segment, in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}, \lambda \in [\lambda_1, \lambda_2]$, where the segment is located between points with position vectors $A\ (\boldsymbol{i} - 5\boldsymbol{j} + 2\boldsymbol{k})$ and $B\ (3\boldsymbol{i} - 2\boldsymbol{k})$.

3. Determine whether the following bodies A and B collide. If they do collide, state the value of the parameter when this takes place and the location.

$\boldsymbol{r_A} = -2\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$ $\qquad$ $\boldsymbol{v_A} = \boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$

$\boldsymbol{r_B} = 6\boldsymbol{i} - 8\boldsymbol{j} + \boldsymbol{k}$ $\qquad$ $\boldsymbol{v_B} = -\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$

4. Determine the position vector of the intersection of the following lines.

$\boldsymbol{r}.(\boldsymbol{i} - 3\boldsymbol{j}) = 4$

$\boldsymbol{r}.(2\boldsymbol{i} + \boldsymbol{j}) = -3$

5. Determine the closest distance between the point with position vector $A\ (-3\boldsymbol{i}+\boldsymbol{j}-6\boldsymbol{k})$ and the line with equation $\boldsymbol{r}=\begin{pmatrix}-1\\4\\-10\end{pmatrix}+\lambda\begin{pmatrix}-1\\-2\\1\end{pmatrix}$, and the value of the parameter when this occurs.

   a. Use a calculus method.

   b. Use a vector method.

6. Two objects A and B are travelling in a line according to the following vector equations.

$\boldsymbol{r_A} = \begin{pmatrix} 3 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\boldsymbol{r_B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Determine the closest distance of separation between the particles and the value of the parameter when this occurs.

a. Use a calculus method.

b. Use a vector method.

7. At 12:00 noon, two ships are moving in straight lines according to the vector equations below. Determine the closest distance between the ships and the time at which this occurs. Units are kilometres and hours.

Ship A: $\boldsymbol{r_A} = \begin{pmatrix} -1 \\ 22 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Ship B: $\boldsymbol{r_B} = \begin{pmatrix} -13 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

a. Use a calculus method.

b. Use a vector method.

8. At 12:00 noon, two ships are moving in straight lines according to the vector equations below. Units are kilometres and hours.

Ship A: $\boldsymbol{r_A} = \begin{pmatrix} -10 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

Ship B: $\boldsymbol{r_B} = \begin{pmatrix} 10 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

a. Determine the closest distance between the ships and the time at which this occurs.

b. Discuss the reasonableness of the results of your calculations in part a.

c. State the closest distance separating the two ships and the time at which this occurs.

9. At 12:00 noon, two objects are moving in straight lines according to the vector equations below. Determine the closest distance between the objects and the time at which this occurs. Units are kilometres and hours.

Object A: $$\boldsymbol{r_A} = \begin{pmatrix} -15 \\ -20 \\ -15 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Object B: $$\boldsymbol{r_B} = \begin{pmatrix} -3 \\ -3 \\ -10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

a. Use a calculus method.

b. Use a vector method.